Northern Beaches Secondary College
Manly Selective Campus

2011
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1- 10
- All questions are of equal value

Marks

Question 1 ***(Answer in a separate booklet)*** **12**

(a) Expand and simplify $2(4x - 1) - 13x$ *(1)*

(b) If the numbers 7, 1 and -5 are the first 3 terms of an arithmetic series, show the nth term is given by $T_n = 13 - 6n$ *(2)*

(c) Solve $|2x - 5| > 3$ and graph the solution on a number line. *(3)*

(d) Differentiate $y = \cos^3 x$. *(2)*

(e) Draw a graph of the function $y = -\sqrt{4 - x^2}$ showing x and y intercepts. *(1)*

(f) Determine the value of a and b if $\dfrac{5}{2 + \sqrt{3}} = a + b\sqrt{3}$ *(2)*

(g) Simplify $\log_5 125$ *(1)*

Marks

Question 2 ***(Answer in a separate booklet)*** **12**

(a) Show that $x^2 - 4x + 2k = 0$ has real roots for $k \leq 2$. *(2)*

(b) Find an indefinite integral of

(i) $\int \left(\sqrt[3]{x} + x^2\right) dx$ *(2)*

(ii) $\int (e^{2x} + \sin 3x)\, dx$ *(2)*

(c) Find the co-ordinates of the vertex and the equation of the directrix for the parabola $y^2 = 8 - 4x$. *(2)*

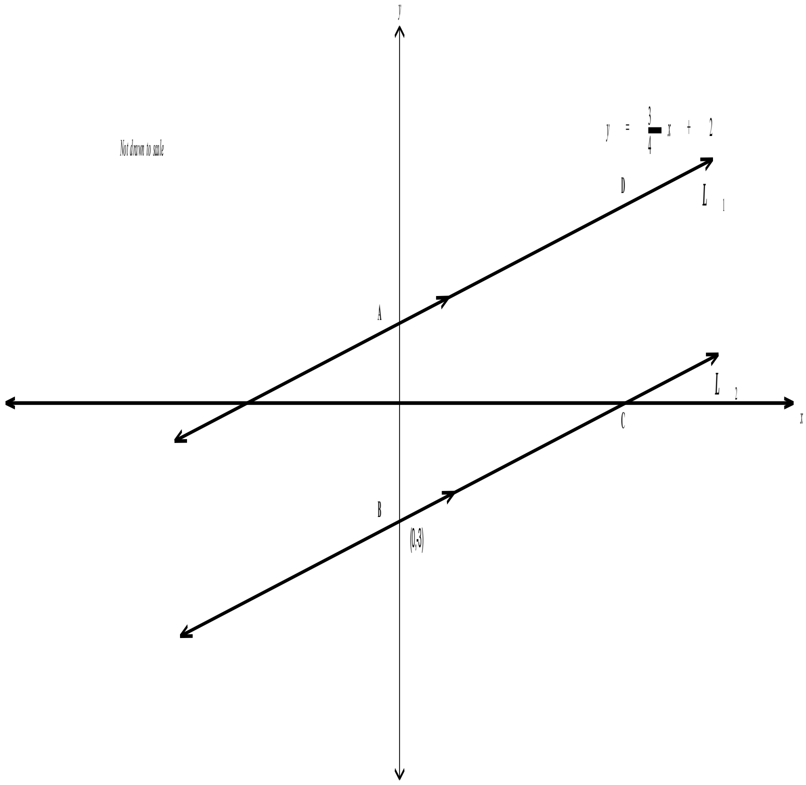
(d) The gradient of the tangent function of a curve is given by $\frac{dy}{dx} = 3 \sin 2x$.
If the curve passes through (0,0), find its equation. *(2)*

(e) Solve $\tan\theta = -2$ for $0 \leq \theta \leq 2\pi$. *(2)*

Marks

Question 3 ***(Answer in a separate booklet)*** **12**

(a)



The line L_1 has equation $y = \frac{3}{4}x + 2$ and passes through the point A on the y-axis.
The line L_2 is parallel to L_1 and passes through B (0,-3).

(i) Write down the co-ordinates of A. ***(1)***

(ii) Find the equation of the line L_2 ***(1)***

(iii) If C is the x-intercept of L_2, write down the co-ordinates of C. ***(1)***

(iv) The point D is positioned on line L_1, such that $ABCD$ is a rhombus.
Find the co-ordinates of D. ***(1)***

(v) Calculate the perpendicular distance from A to L_2. ***(1)***

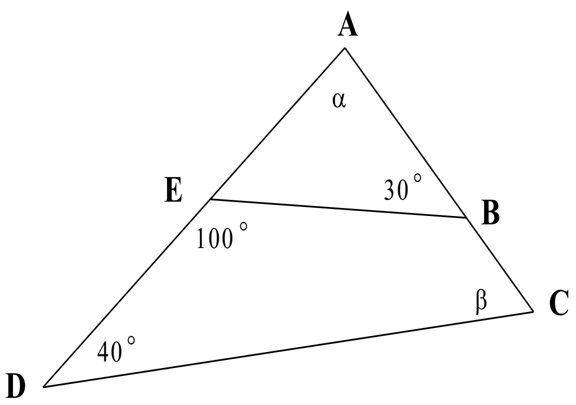
(vi) Hence, or otherwise, calculate the area of the rhombus $ABCD$. ***(2)***

Question 3 continues on the next page.

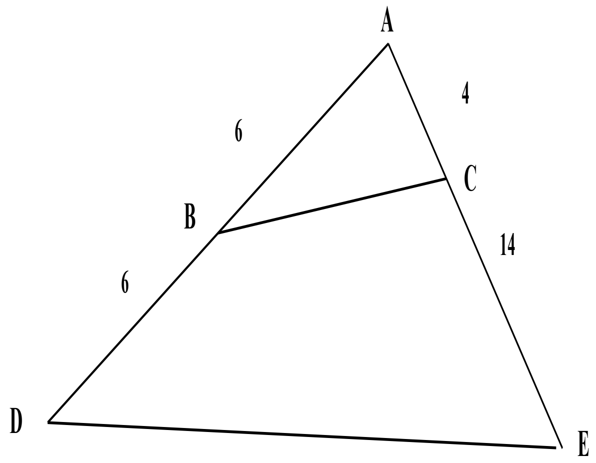
Marks

Question 3 (continued)

(b) Calculate the size of angles α and β giving reasons for your answer. ***(2)***



(c) Prove that $\Delta BAC \,|||\, \Delta DAE$ ***(3)***



Marks

Question 4 ***(Answer in a separate booklet)*** ***12***

a) Determine the value of A, B and C in the following. ***(3)***

$$2x^2 + 6x + 9 \equiv A(x+2)(x-1) + B(x+2) + C$$

b) The first three terms of a series are 3, 6 and 12 *(leave your answers in index form)*.

(i) What is the value of T_8? ***(1)***

(ii) What is the sum of the first fifteen terms? ***(1)***

c) The first multiple of seven after 130 is 133. What is the sum of all the multiples of seven between 130 and 500? ***(4)***

d) What is the maximum value for S_n? ***(3)***

$$S_n = 1 + \frac{\sin\theta}{2} + \frac{\sin^2\theta}{4} + \frac{\sin^3\theta}{8} + \ldots$$

Marks

Question 5 ***(Answer in a separate booklet)*** **12**

(a) The quadratic equation $2x^2 - 3x + 6 = 0$ has roots α and β.
Find the value of:

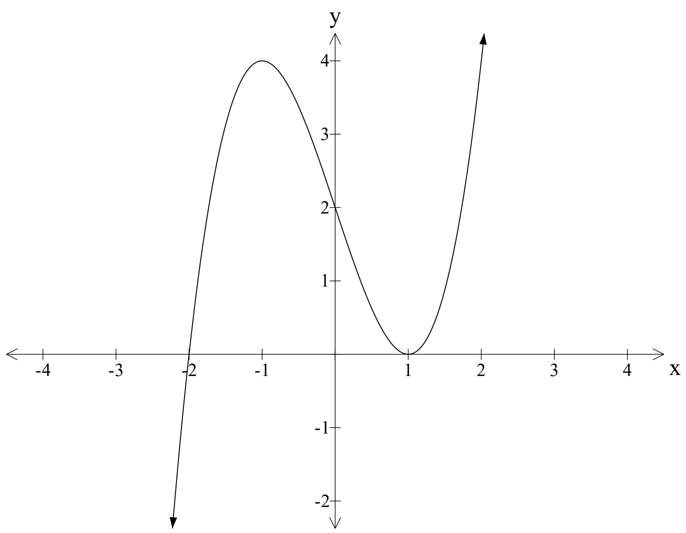
(i) $\alpha + \beta$ *(1)*

(ii) $\alpha\beta$ *(1)*

(iii) $\alpha^2 + \beta^2$ *(2)*

(b) Differentiate $y = \sin 2x^{\circ}$. *(2)*

(c) Copy the graph shown below into your answer booklet and, on the same set of axes, sketch the curve of the derivative function. *(2)*



(d) Consider the curve $y = 12x^3 - 3x^4$.

Find the stationary points and determine their nature. *(4)*

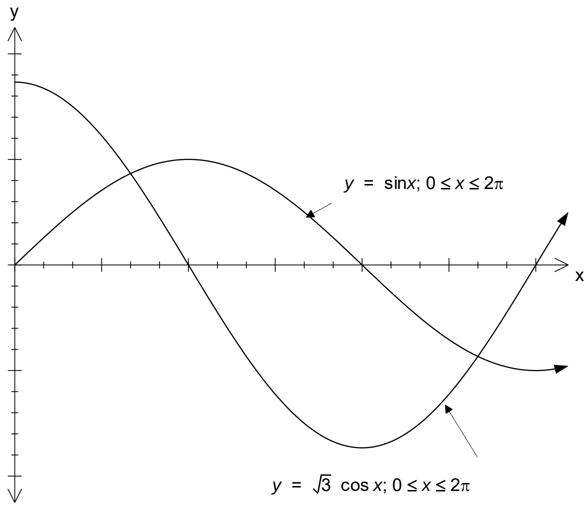
Marks

Question 6 ***(Answer in a separate booklet)*** **12**

(a) (i) State the period of the equation $y = 3 - \cos 2x$. *(1)*

(ii) Graph $y = 3 - \cos 2x$ for $0 \le x \le \pi$. *(3)*

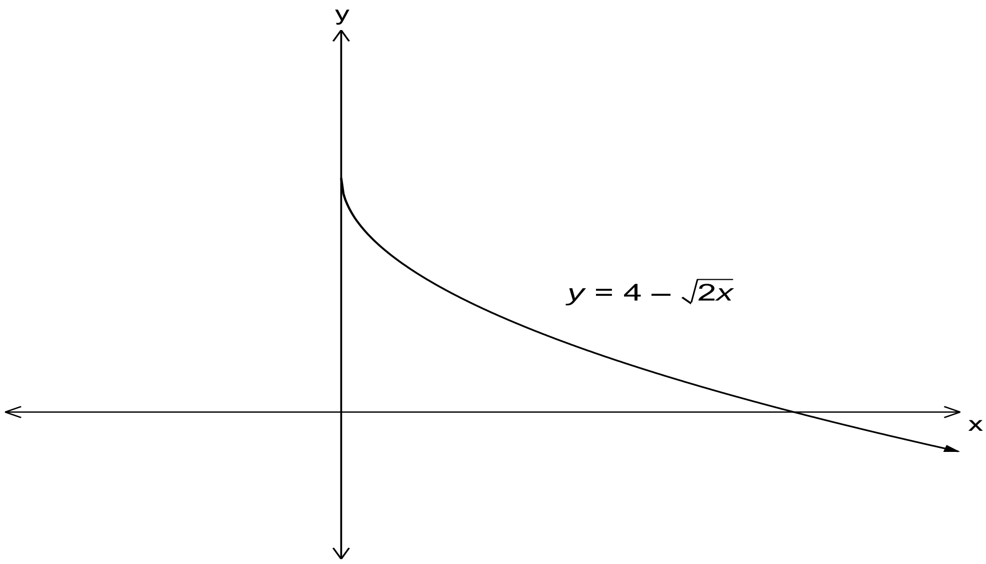
(b)



(i) Show that the curves $y = \sin x$ and $y = \sqrt{3}\cos x$ intersect at $x = \frac{\pi}{3}$ and $x = \frac{4\pi}{3}$. *(1)*

(ii) Hence determine the area enclosed between the curves for $0 \le x \le 2\pi$. *(3)*

(c) The diagram below shows the function $y = 4 - \sqrt{2x}$.



If the area enclosed by this curve $y = 4 - \sqrt{2x}$, the x-axis and the y-axis is rotated about the y-axis, find the volume of the solid of revolution formed. *(4)*

Marks

Question 7 ***(Answer in a separate booklet)*** **12**

(a) The mass M kg of a radioactive substance present after t years is given by

$$M = 10e^{-kt}$$

where k is a positive constant. After 100 years the mass has been reduced to 5 kg.

(i) What was the initial mass? *(1)*

(ii) Find the value of k. *(1)*

(iii) What amount of radioactive substance would remain after a period of 1000 years? *(1)*

(iv) How long would it take for the initial mass to reduce to 8kg? *(2)*

(b) A student attaches an air pressure pump to his bike tyre to inflate the tyres. The rate of change of the pressure in the tyre is given by

$$\frac{dP}{dt} = \frac{t}{2} - \frac{1}{8} \text{ units of pressure/sec}$$

where t is the time after pumping has started.

After 4 seconds of pumping, the pressure in the tyre is 15 units.

(i) What is the initial pressure in the tyre? *(2)*

(ii) The bike tyre will burst when the pressure reaches 40 units of pressure. At what time does this occur? *(answer to the nearest second)*. *(2)*

(c) Determine the values of x in the following equation. *(3)*

$$3 - 4\cos^2(2x) = 0 \qquad 0 \le x \le \pi$$

Marks

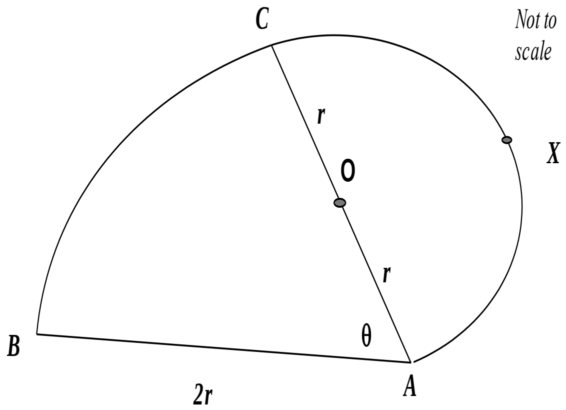
Question 8 ***(Answer in a separate booklet)*** **12**

(a) *(i)* A is the point (1, 1) and B is the point (4, 1). A point P (x, y) moves such that $PA \perp PB$.

Show that the locus of P is given by $x^2 - 5x + y^2 - 2y = -5$. ***(3)***

(ii) Give a detailed geometric description of the locus in part (i). ***(2)***

(b) The cross-section of a metal object is formed as shown in the figure below.



The cross-section consists of a semi-circle AXC, centred at O and radius r, and a sector ABC of radius $2r$, centred at A with angle θ.

(i) What is the perimeter $AXCB$ of the object in terms of r and θ? ***(2)***

(ii) If the area of the cross-section is fixed at 4 square units, show that the perimeter in part (i) can also be expressed as ***(2)***

$$P = \frac{4}{r} + r\left(2 + \frac{\pi}{2}\right)$$

(iii) Show that the perimeter in part (ii) is smallest when $r^2 = \dfrac{8}{4 + \pi}$. ***(3)***

Marks

Question 9 ***(Answer in a separate booklet)*** **12**

(a) The volume formed by rotating the curve $y = \log_e x$ about the x – axis between $x = 2$ and $x = 6$ can be expressed as:

$$\text{Volume} = \pi \int_2^6 (\log_e x)^2 \, dx$$

Use Simpson's Rule with three function values to find an approximation for the volume of the solid of revolution formed. *(3)*

(b) A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by

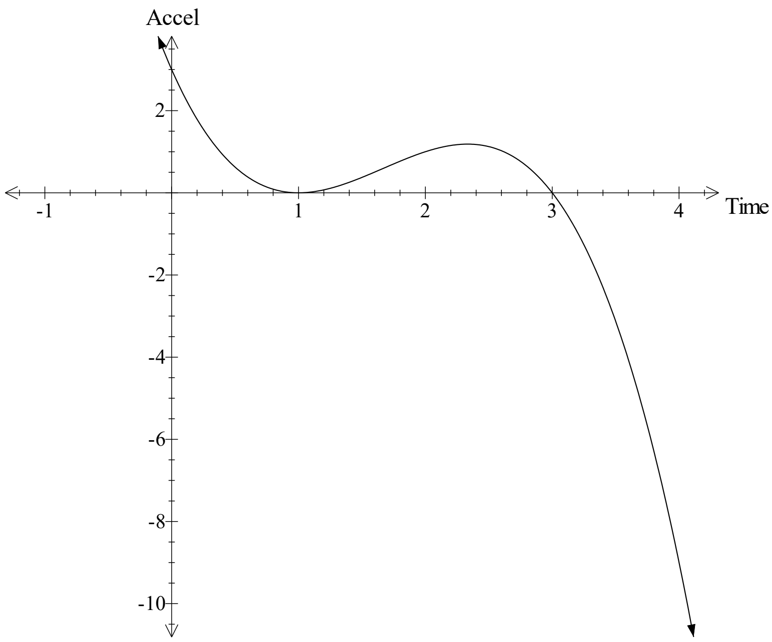
$$x = 4 + 3t\,e^{-2t}$$

(i) Show the velocity of the particle is given by $\dot{x} = 3e^{-2t}(1 - 2t)$. *(2)*

(ii) Show the particle is at rest when $t = \frac{1}{2}$. *(1)*

(iii) Find the greatest possible total distance the particle could travel. *(3)*

(c) An object is moving on the x – axis. The graph below shows the acceleration $a = \frac{d^2x}{dt^2}$.



Draw a velocity-time graph for $0 \leq t \leq 4$ if the object is initially travelling to the left with a speed of 3 m/s. *(3)*

Marks

Question 10 ***(Answer in a separate booklet)*** **12**

Consider the function y = $(x - 1)\log_e 2x$.

(i) What is the domain of this function? *(1)*

(ii) Find the x-intercepts of the function. *(2)*

(iii) Show that the gradient of the curve is given by $\frac{dy}{dx} = \frac{x-1}{x} + \log_e 2x$. *(1)*

(iv) Determine the gradient of the function at $x = \frac{1}{2}$ and at $x = 1$. *(2)*

(v) Without further calculation, determine whether the function has a maximum or minimum value. Justify your conclusion. *(2)*

(vi) By using any of the results above, show that $\int_{\frac{1}{2}}^{1} \log_e 2x \, dx = \log_e 2 - \frac{1}{2}$ *(4)*

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

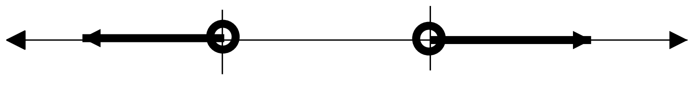
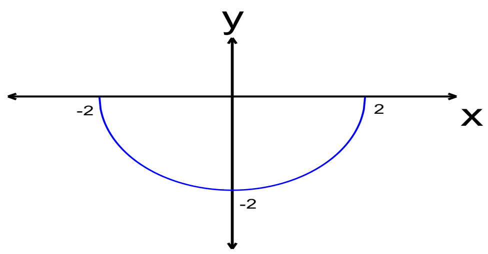
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

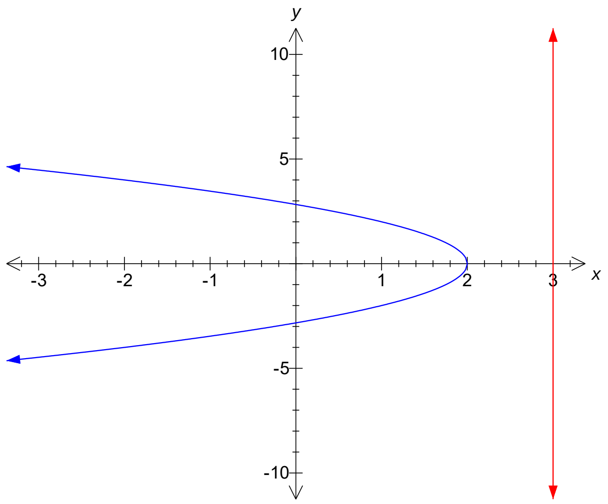
(*2)*

Manly Selective Campus
2011 HSC Mathematics Trial - solutions

Question 1

a)	$2(4x - 1) - 13x = 8 - 2 - 13x$ $= -5x - 2$	**1** correct solution 0 marks if then solved as an equation
b)	7, 1, -5 $d = 1 - 7 \quad d = -5 - 1 \quad a = 7$ $= -6 \quad = -6$ $T_n = a + (n - 1)d$ $= 7 + (n - 1) \times -6$ $= 7 - 6n + 6$ $= 13 - 6n$ (*as required*)	**2** correct solution **1** either showing $T_n = 7 + (n - 1) \times -6$ **OR** $d = -6$
c)	$\lvert 2x - 5 \rvert > 3$ $2x - 5 > 3$ or $-(2x - 5) > 3$ $2x > 8 \quad -2x + 5 > 3$ $x > 4 \quad -2x > -2$ $x < 1$  	**3** solution correct **2** both solutions but no number line **1** only one correct solution
d)	$y = \cos^3 x$ $= (\cos x)^3$ $\frac{dy}{dx} = 3(\cos x)^2 \times -\sin x$ $= -3\sin x \cos^2 x$	**2** correct solution **1** any one of $-$, 3, $\sin x$ or $\cos^2 x$ absent from correct solution
e)	 	**1** correct graph showing both x and y intercepts
f)	$\frac{5}{2 + \sqrt{3}} = a + b\sqrt{3}$ LHS $= \frac{5}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$ $= \frac{!0 - 5\sqrt{3}}{4 - 3}$ $= 10 - 5\sqrt{3} \quad \therefore$ a = 10, b = -5	**2** both a and b correct **1** multiplying numerator and denominator by $2 - \sqrt{3}$
g)	$\log_5 125 = \log_5 5^3$ $= 3\log_5 5$ $= 3 \times 1$ $= 3$	**1** correct solution

Question 2

(a)	For real roots, $\Delta \ge 0$ $(-4)^2 - 4\times 2k \ge 0$ $16 - 8k \ge 0$ $-8k \ge -16$ $k \le 2$ as required	2 marks correct solution 1 mark for getting to line 3 Note: taking a few particular values of *k* and substituting in is not a general proof
(b) (i)	$\int x^{\frac{1}{3}} + x^2\,dx = \frac{3}{4}x^{\frac{4}{3}} + \frac{x^3}{3} + c$ or $\frac{3}{4}x\sqrt[3]{x} + \frac{x^3}{3} + c$	*2 marks- correct integral including +c* *1 mark – 1 error in answer* Note: Many students need to revise index laws so the can rewrite $\sqrt[3]{x}$ correctly in index form
(ii)	$\frac{1}{2}e^{2x} - \frac{1}{3}\cos 3x + c$	*2 marks – correct answer* *1 mark – one integral correctly found* Note: Students should use the Table of Standard Integrals to correctly fine co-efficients
(c)	Rewrite given equation as $y^2 = -4(x-2)$ is of the form $(y-k)^2 = -4a(x-h)$ *where* $a = 1$ Vertex is $(h,k) = (2,0)$  Directrix is $x=3$	*2 marks for correct vertex and directrix equation* *1mark for correct vertex OR correct directrix from incorrect vertex* NOTE!!!!: the focal length is a LENGTH so stating that *a*= -1 (as many did!!) is VERY WRONG. Main other error was in not rearranging terms to correct form which requires *x* to be first term in bracket.
(d)	$\frac{dy}{dx} = 3\sin 2x$ $y = -\frac{3}{2}\cos 2x + c$ $(0,0)$ $\quad 0 = -\frac{3}{2}\cos 0 + c \quad \therefore c = \frac{3}{2}$ $\therefore \quad y = -\frac{3}{2}\cos 2x + \frac{3}{2}$	*2 marks- correct solution* *1 mark correct Line 2 OR incorrect integral but correct finding of constant from error.*
(e)	$\tan\theta = -2$ Related acute $= 63^\circ 26' = 1.107148718^c$ $\theta = \pi - 1.107148718^c$ or $\theta = 2\pi - 1.107148718^c$ $\theta \approx 2.03$ or 5.18	*2 marks – correct solutions, including reasonable approximations in terms of* π *1 mark – correct answer in degrees OR 1 correct answer in radians OR correct radians but in first or third quadrants OR correct radian answers from making -2 the radians*

Question 3

(ai)	(0,2)	1 mark – correct answer must STATE CO-ORDINATES
(aii)	$y=\frac{3}{4}x-3$	1 mark – correct answer
(aiii)	(4,0)	1 mark – correct answer
(aiv)	*CD* must be same length as *AB* and parallel to it, so *D* is (4,5)	1 mark – correct answer
(av)	$\mathbf{L}_2: 3x-4y-12=0\ \ A(0,2)$ $d_\perp=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $d_\perp=\frac{\lvert 3\times 0-4\times 2-12\rvert}{\sqrt{3^2+4^2}}$ $d_\perp=\frac{\lvert -20\rvert}{5}$ $d_\perp=4$	1 mark – correct answer
(avi)	A rhombus is also a parallelogram $A=bh_\perp$ $=5\times 4$ $=20$ *square units*	2 marks – correct area or correct from error in (v) 1 mark – correct length of BC or AD
(b)	100= α +30(exterior angle of triangle *AEB* equals sum of interior opposite angles) $\alpha=70$ 40+70+ β =180(angle sum of triangle *ACD*) $\beta=70$	2 marks – correct angles with correct reasons given for the approach taken 1 mark – correct angle with correctly argued reasons
(c)	In ΔBAC and ΔDAE, $\frac{AB}{AE}=\frac{6}{18}=\frac{1}{3}$ $\frac{AC}{AD}=\frac{4}{12}=\frac{1}{3}\quad\therefore\frac{AB}{AE}=\frac{AC}{AD}$ $\angle BAC$ is a common angle $\therefore\Delta BAC\ \lvert\lvert\lvert\ \Delta DAE$(two pairs of corresponding sides in proportion about the same INCLUDED angle)	3 marks – correctly argued proof 2 marks – either omitting test used to prove similarity OR incorrect/inadequate wording of test OR not directly identifying names of corresponding sides (only referring to lengths) OR only giving test reason and omitting which triangle are similar 1 mark – finding common angle OR giving numerical values of correct corresponding sides

Question 4

(a)	$2x^2 + 6x + 9 \equiv A(x+2)(x-1) + B(x+2) + C$ *coefficient of* x^2 $A = 2$ *let x= -2* $2 \times (-2)^2 + 6 \times (-2) + 9 = 0 + 0 + C$ $C = 5$ *let x= 1* $2 \times 1^2 + 6 \times 1 + 9 = 0 + 3B + 5$ $B = 4$	*3 marks – one mark for each correct answer.*
(b) (i)	$T_1 = 3 \quad T_2 = 6 \quad T_3 = 12$ $\frac{T_2}{T_1} = \frac{T_3}{T_2} = 2 = r$ $T_8 = ar^{n-1}$ $= 3 \times 2^7$	*1 mark – correct answer*
(b) (ii)	$T_{15} = 3 \times 2^{14}$ $S_{15} = \frac{a(r^n - 1)}{r - 1}$ $= \frac{3(2^{15} - 1)}{2 - 1}$ $= 98301$	*1mark – correct answer – non-index form accepted.*
(c)	$T_1 = 133$ $Tn = 133 + (n-1) \times 7$ $= 126 + 7n$ $126 + 7n < 500$ $7n < 374$ $n < 53.43$ $n = 53$ $T_{\lambda st} = 126 + 7 \times 53$ $= 497$ $S_n = \frac{n}{2}(a + l)$ $= \frac{53}{2}(133 + 497)$ $= 16695$	*4 marks – correct solution* *3 marks – correct value for last term in series* *2 marks – correct value for n* *1 mark – expression for T_n*

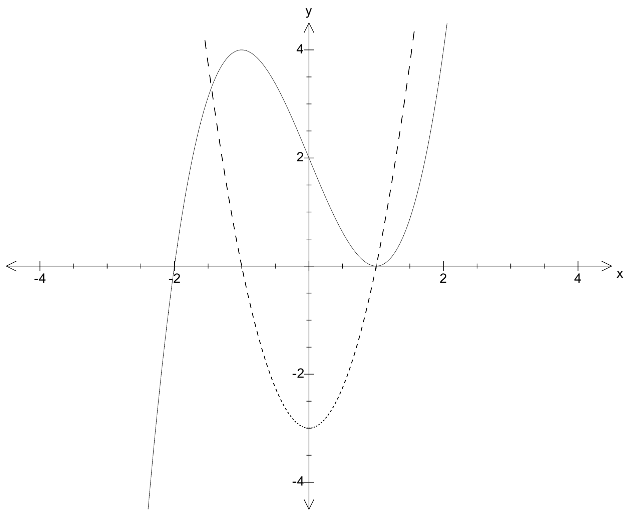
Question 4 (continued)

(d)	$S_n = 1 + \frac{\sin\theta}{2} + \frac{\sin^2\theta}{4} + \frac{\sin^3\theta}{8} + \ldots$ *Geometric Series* $r = \frac{\sin\theta}{2}$ $-1 \le \sin\theta \le 1$ $\therefore -\frac{1}{2} \le \frac{\sin\theta}{2} \le \frac{1}{2}$ $\therefore$ *Series will have limiting sum.* $Sn = \frac{a}{1-r}$ $= \frac{1}{1-\frac{\sin\theta}{2}}$ S_n *is maiximum when denominator is min. ie.* $\frac{\sin\theta}{2}$ *is at its max value* $Sn = \frac{1}{1-\frac{1}{2}}$ $= 2$	*3 marks – correct answer.* *2 marks – correct expression for* S_n *1 mark – identify series as geometric series and determining value of r will mean that it has a limiting sum.*

Question 5

<table>
<tr><td>(a)(i)</td><td>$a = 2 \quad b = -3 \quad c = 6$
$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{2} = \frac{3}{2}$</td><td>*1 mark – correct answer*</td></tr>
<tr><td>(ii)</td><td>$\alpha\beta = \frac{c}{a} = 3$</td><td>*1 mark – correct answer*</td></tr>
<tr><td>(iii)</td><td>$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= \frac{9}{4} - 6 = -\frac{15}{4}$</td><td>*2 marks – correct answer*

1 mark correct expression</td></tr>
<tr><td>(b)</td><td>*Cannot use calculus on trig functions expressed in degrees – must be radians.*
$y = \sin 2x^\circ = \sin \frac{2\pi}{180} x$
$\therefore \frac{dy}{dx} = \frac{2\pi}{180} \cos \frac{2\pi}{180} x = \frac{\pi}{90} \cos \frac{\pi x}{90}$</td><td>*2 marks – correct derivative*

1 mark – recognition of need for radians
NOTE: When using calculus with trig functions, you must use RADIANS.</td></tr>
<tr><td>(c)</td><td>
</td><td>*2 marks – correct graph*

1 mark – correct x-intercepts

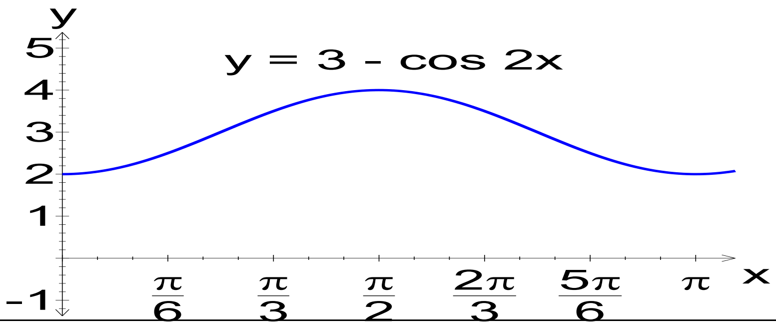
In many diagrams, the two curves were hard to distinguish. Use different colours ,dotted lines, etc to identify each and label.</td></tr>
<tr><td>(d)</td><td>$y = 12x^3 - 3x^4$
$\frac{dy}{dx} = 36x^2 - 12x^3$
For SP $\frac{dy}{dx} = 0$
$\therefore \; 12x^2(3 - x) = 0$
$x = 0 \text{ or } x = 3$
$\frac{d^2y}{dx^2} = 72x - 36x^2$
At $x = 0 \; \frac{d^2y}{dx^2} = 0$ so possible POI
At x = 3 $\frac{d^2y}{dx^2} = -108 < 0 \therefore$ max
$\therefore$ SP at (0, 0) POI
(3, 81) maximum value
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>y''</td><td>-108</td><td>0</td><td>36</td></tr>
</table></td><td>*4 marks – correct points and nature determined*

3 marks – correct points but incomplete analysis of second derivative or no y values.

2 marks – correct first derivative and x values determined

1 mark – correct derivatives</td></tr>
</table>

Question 6

a) i)	i) period $= \frac{2\pi}{2}$ $= \pi$	**1** correct answer
ii)	 	**3** correct graph **2** $y = -\cos 2x$ at start other than 2 on y-axis **or** $y = 3 + \cos 2x$ **or** $y = 2 - \cos x$ **1** $y = \cos 2x$
b) i)	$x = \frac{\pi}{3}$ $\quad y = \sin\left(\frac{\pi}{3}\right)$ $\quad\quad y = \sqrt{3}\cos\left(\frac{\pi}{3}\right)$ $= \sqrt{\frac{3}{2}}$ $\quad\quad = \sqrt{\frac{3}{2}}$ $x = \frac{4\pi}{3}$ $\quad y = \sin\left(\frac{4\pi}{3}\right)$ $\quad\quad y = \sqrt{3}\cos\left(\frac{4\pi}{3}\right)$ $= -\sin\left(\frac{\pi}{3}\right)$ $\quad\quad = -\sqrt{3}\cos\left(\frac{\pi}{3}\right)$ $= -\sqrt{\frac{3}{2}}$ $\quad\quad = -\sqrt{\frac{3}{2}}$ **OR** $\sqrt{3}\cos x = \sin x$ $1 = \sin\frac{x}{\sqrt{3}\cos x}$ $1 = \frac{1}{\sqrt{3}}\tan x$ $\tan x = \sqrt{3}$ $\quad$ 1st quadrant $\angle\ \frac{\pi}{3}$ $\tan x > 0$ *quad* 1 and 3 $x = \frac{\pi}{3}, \frac{4\pi}{3}$	**1** showing intersection at both points **OR** using tan to find the 2 solutions

Question 6 (continued)

ii)	$Area = \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \sqrt{3}\cos x)\,dx$ $= [-\cos x - \sqrt{3}\sin x]$ $= \left(-\cos\left(\frac{4\pi}{3}\right) - \sqrt{3}\sin\left(\frac{4\pi}{3}\right)\right) - \left(-\cos\left(\frac{\pi}{3}\right) - \sqrt{3}\sin\left(\frac{\pi}{3}\right)\right)$ $= \left(- -\frac{1}{2} - \sqrt{3} \times -\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} - \sqrt{3} \times \frac{\sqrt{3}}{2}\right)$ $= \left(\frac{1}{2} + \frac{3}{2}\right) - \left(-\frac{1}{2} - \frac{3}{2}\right)$ $= \left(\frac{4}{2}\right) - \left(-\frac{4}{2}\right)$ $= 2 + 2$ $Area\ is\ 4\ units^2$	**3** correct solution **2** 1of the integral sections correct with correct solution for the incorrect integral **1**1 correct integral only **OR** 2 integrals incorrect with incorrect limits **OR** area broken into 3 regions with correct answer of 4 units2 for the only correct region
c)	y intercept when $x = 0\ y = 4$ $y = 4 - \sqrt{2x}$ $\sqrt{2x} = 4 - y$ $2x = (4 - y)^2$ $x = \frac{(4 - y)^2}{2}$ $x^2 = \frac{(4 - y)^4}{4}$ $\text{Volume} = \pi\int_0^4 \frac{(4 - y)^4}{4}\,dy$ $= \frac{\pi}{4}\left[\frac{(4 - y)^5}{5 \times -1}\right]_4^0$ *** $= -\frac{\pi}{20}\left\{(4 - 4)^5 - (4 - 0)^5\right\}$ $= -\frac{\pi}{20} \times -1024$ $= \frac{256\pi}{5}\ units^3$	**4**correct solution **3** correct integral to *** **OR** no pi in solution. **2** correct value to x^2 1correct y intercept

Question 7

(a)- (i)	$M = 10e^{-kt}$ At $t = 0$ $M = 10e^{-k \times 0}$ $M = 10$ therefore initial mass is 10kg	*1 mark – correct answer*
(ii)	$M = 10e^{-kt}$ at $t = 100$ $m = 5$ $5 = 10e^{-100k}$ $\dfrac{\ln\left(\frac{1}{2}\right)}{-100} = k$ $k = \dfrac{\ln 2}{100} = 6.93 \times 10^{-3}$	*1 mark – correct answer*
(iii)	$M = 10e^{-1000k}$ $= 9.765 \times 10^{-3}$ ***kg***	*1 mark – correct answer*
(iv)	$8 = 10e^{-kt}$ $t = \dfrac{\ln(0.8)}{-k}$ $t = 32.19$ *year*	*2 mark – correct answer* *1mark – correct approach with arithmetic error*
(b)-(i)	$\dfrac{dP}{dt} = \dfrac{t}{2} - \dfrac{1}{8}$ $P = \dfrac{t^2}{4} - \dfrac{t}{8} + C$ $t = 4$ $P = 15$ $15 = \dfrac{4^2}{4} - \dfrac{4}{8} + C$ $C = 11.5$ $P = \dfrac{t^2}{4} - \dfrac{t}{8} + 11.5$ At $t = 0$ $P = 11.5$ Therefore initial pressure is 11.5 units.	*2 marks – correct answer* *1 mark – correct integration with C*

Question 7 (continued)

(b)-(ii)	$P = \frac{t^2}{4} - \frac{t}{8} + 11.5$ $40 = \frac{t^2}{4} - \frac{t}{8} + 11.5$ $320 = 2t^2 - t + 92$ $t = \frac{1 \pm \sqrt{1825}}{4} = 10.9 \text{ sec} \approx 11 \text{ sec}$ *as* $t \geq 0$	*2 marks – correct answer* *1 mark – correct approach reaching two possible answers (one neg and one positive) with one arithmetic error.*
(c)	$3 - 4\cos^2(2x) = 0$ $0 \leq x \leq \pi$ $\cos(2x) = \pm\frac{\sqrt{3}}{2}$ $0 \leq 2x \leq 2\pi$ $2x = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{12}, \frac{\pi}{12}, \frac{2\pi}{12}, \frac{5\pi}{12}$	*3 marks – correct answer* *2 marks – incomplete domain* *1 mark – failure to recognise ±*

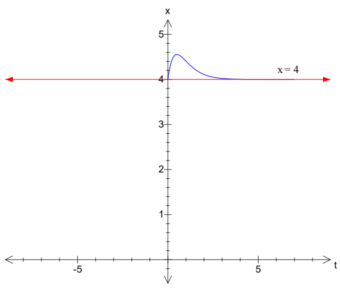
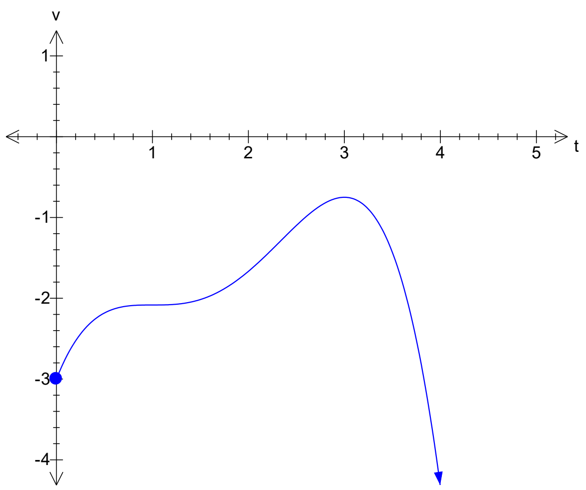
Question 8

(a) (i)	$m_{PA} \times m_{PB} = -1$ $\frac{y-1}{x-1} \times \frac{y-1}{x-4} = -1$ $(y-1)^2 = -(x-1)(x-4)$ $y^2 - 2y + 1 + (x-1)(x-4) = 0$ $y^2 - 2y + 1 + x^2 - 5x + 4 = 0$ $x^2 - 5x + y^2 - 2y = -5$	*3 marks – correct demonstration* *2 marks – product of gradients expressed correctly but subsequent error* *1 mark – both gradients correct*
(ii)	Completing the square: $\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + (y-1)^2 - 1 + 5 = 0$ $\left(x - \frac{5}{2}\right)^2 + (y-1)^2 = \left(\frac{3}{2}\right)^2$ The locus is a circle centred at $\left(\frac{5}{2}, 1\right)$ with radius $\frac{3}{2}$	*2 marks – complete description* *1 mark – squares completed correctly.*
(b) (i)	Perimeter of semi-circle AXC = πr CB Arc length = $2r\theta$ Hence perimeter AXCB = $\pi r + 2r\theta + 2r$	*2 marks – correct perimeter* *1 mark –at least one of the first two quantities correct*
(bii)	$Area = A_{\text{sector}} + A_{\text{semi circle}}$ $4 = \frac{1}{2}(2r)^2\theta + \frac{1}{2}\pi r^2$ $4 = 2r^2\theta + \frac{1}{2}\pi r^2$ $2r^2\theta = 4 - \frac{1}{2}\pi r^2$ $\theta = \frac{2}{r^2} - \frac{\pi}{4}$ $\therefore \quad P = 2r + \pi r + 2r\left(\frac{2}{r^2} - \frac{\pi}{4}\right)$ $P = 2r + \pi r + \frac{4}{r} - \frac{\pi r}{2}$ $P = \frac{4}{r} + 2r + \frac{\pi r}{2}$ $P = \frac{4}{r} + r\left(2 + \frac{\pi}{2}\right)$	*2 marks – correct proof* *1 mark – correct expression for θ substituted into P*

Question 8 (continued)

(iii)	$P = 4r^{-1} + 2r + \frac{\pi r}{2}$ $\frac{dP}{dr} = -4r^{-2} + 2 + \frac{\pi}{2}$ $\frac{d^2P}{dr^2} = 8r^3 > 0$ for all r>0 tf minimum For min $\frac{dP}{dt} = 0$ $\therefore \frac{4}{r^2} = 2 + \frac{\pi}{2}$ $r^2 = \frac{8}{4+\pi}$	*3 marks – correct and complete demonstration* *2 marks – correct derivative and expression for* r^2 *but subsequent error or no proof of minimum* *1 mark – correct derivative, no proof of a minimum or algebraic error.*

Question 9

(a)	Vol $= \pi \int_2^6 (\log_e x)^2\, dx \approx \frac{(6-2)\pi}{6}[(\ln 2)^2 + 4(\ln 4)^2 + (\ln 6)^2]$ $= \frac{2\pi}{3}$ x 11.37810323 $= 23.83\, u^3$	3 marks – correct solution 2 marks – correct Simpson's rule except for loss of π or incorrect or omitted y^2 or calculation error 1 mark – a correct use of Simpson's rule on $\int_2^6 \log_e x\, dx$ NOTE: many students misinterpreted $(\log_e x)^2$
(b) (i)	$\dot{x} = 3t.\,-2e^{-2t} + e^{-2t}.\,3$ $= 3e^{-2t}(1-2t)$	2 marks for correct velocity expression 1 mark for correct application of product rule
(b) (ii)	Either substitute $t = \frac{1}{2}$, $3e^{-2\text{x}\frac{1}{2}}\left(1 - 2\text{x}\frac{1}{2}\right) = 0$ i.e. particle is at rest. OR Solve $e^{-2t}(1-2t) = 0$ $\therefore e^{-2t} = 0$ or $1 - 2t = 0$ $e^{-2t} \neq 0$ as $e^{-2t} > 0$ $t = \frac{1}{2}$ as req'd	1 mark – either correct solution NOTE: many students either did not write a conclusion for the substitution approach OR ignored the possibility of $e^{-2t} = 0$ OR omitted a justification for not solving
(b) (iii)	When $t=0$, $x=4$ When $t=0.5$, $x = 4 + \frac{3}{2e}$ As $t \to \infty$, $e^{-2t} \to 0 \therefore x \to 4$  Max distance $= 2 \text{ x } \frac{3}{2e} = \frac{3}{e}$ metres	3 marks – correct solution 2 marks – establishing starting position, maximum and limit 1 mark – finding max displacement NOTE: many students believed the max displacement represented the greatest possible total distance
(c)	Graph had to start at (0,-3), have a point of inflection when $t = 1$, and a rel max at $t = 3$  	3 marks – correct graph 2 marks – 1 error 1 mark - 2 errors

Question 10

(a) (i)	$y = (x\text{-}1)\log_e 2x$ Domain of (x-1) is all real x while the domain of ln x is x>0. Hence the domain of the product is x > 0.	*1 mark –correct domain*
(ii)	x-intercepts when y = 0. Hence $x = 1$ OR $\ln 2x = o$ So $x = 0.5$	*2 marks – both intercepts correct* *1 mark – one intercept correct*
(iii)	$\frac{dy}{dx} = \ln 2x \times 1 + (x-1) \times \frac{1}{x}$ $= \frac{x-1}{x} + \ln 2x$	*1 mark –correct derivative*
(iv)	$At\ x = \frac{1}{2},\ \frac{dy}{dx} = \frac{-\frac{1}{2}}{\frac{1}{2}} + \ln 2 \times \frac{1}{2} = \text{-}1$ At x = $1\frac{dy}{dx} = \ln 2$	*2 marks – both gradients correct* *1 mark – one gradient correct.*
(v)	The curve has a minimum as the gradient changes from negative to positive and is continuous throughout the sub-domain stated.	*2 marks - statement of a minimum supported with both statements* *1 mark statement of a minimum with some appropriate support*
(vi)	From part (iii) $\frac{d}{dx}\{(x-1)\ln 2x\} = 1 - \frac{1}{x} + \ln 2x$ $\therefore \int_{\frac{1}{2}}^{1} \frac{d}{dx}\{(x-1)\ln 2x\} = \int_{\frac{1}{2}}^{1} 1dx - \int_{\frac{1}{2}}^{1} \frac{1}{x}dx + \int_{\frac{1}{2}}^{1} \ln 2x\, dx$ $\left[(x-1)\ln 2x\right]_{\frac{1}{2}}^{1} = \left[x - \ln x\right]_{\frac{1}{2}}^{1} + \int_{\frac{1}{2}}^{1} \ln 2x\, dx$ $\therefore \int_{\frac{1}{2}}^{1} \ln 2x\, dx = (0-0) - \left[(1-0) - \left(\frac{1}{2} - \ln\frac{1}{2}\right)\right]$ $= -\frac{1}{2} - \ln 1 + \ln 2 = \ln 2 - \frac{1}{2}$	*4 marks – correct process to demonstrate required result* *3 marks – correct up to final substitution* *2 marks – integral and limits mostly correct* *1 mark – correct setting out for integral using the derivative*